# WEBINAR MATEMATIK TAMBAHAN:

# PROBABILITY DISTRIBUTION

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"Success is not final,
failure is not fatal: it is
the courage to
continue that counts"

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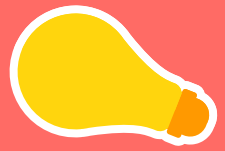


# RANDOM VARIABLE



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# LEARNING STANDARDS

- Describe the meaning of random variable.
- Compare and contrast discrete random variable and continuous random variable.
- Describe the meaning of probability distribution for discrete random variables.
- Construct table and draw graph of probability distribution for discrete random variable.

# 5.1.1 Random variable

**A RANDOM VARIABLE IS A VARIABLE WITH NUMERIC VALUES THAT CAN BE DETERMINED FROM A RANDOM PHENOMENON**

## Discrete random variable

-Random variable that have **countable numbers of values**

-Usually taking values like **zero and positive integers**

-Example: X is a random variable which represent number of white car among 3 cars in the parking lot

- Written as X = {0,1, 2, 3 }

## Continuous random variable

-Random variable that **are not integers**

-Take values that **lies in interval**

-Example: Y represent the weight in kg obtained from students at MRSM P.

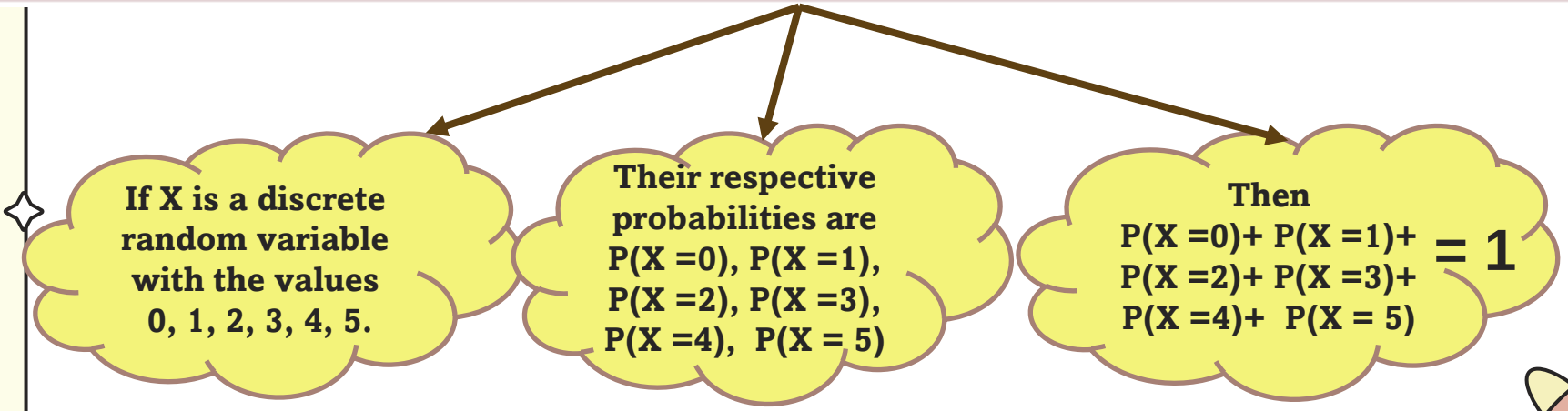
-Written as Y = $\{40 \leq y \leq 80\}$

# 5.1.2 Discrete random variable and continuous random variable

| SITUATION | Type of random variable | written as |
|---|---|---|
| A fair dice is thrown three times, given X is a random variable which represents the number of times to get the number 4 | DISCRETE | X = { 0, 1, 2, 3 } |
| The shortest building in Seroja city is 3 m while the tallest is 460 m. X represents the heights of the buildings located in the city of Seroja. | CONTINUOUS | X = { $3 \leq x \leq 460$} |
| Six prefects are randomly selected from pupils of Form 5. X represents the number of prefects who wear glasses. | DISCRETE | X = { 0, 1, 2, 3, 4, 5, 6 } |

# Probability distribution for discrete random variables

If $X$ is a discrete random variable with the values $r_1, r_2, r_3, \ldots, r_n$ and their respective probabilities are $P(X = r_1), P(X = r_2), P(X = r_3), \ldots, P(X = r_n)$, then $\sum_{i=1}^{n} P(X = r_i) = 1$, thus each $P(X = r_i) \geqslant 0$.



# EXAMPLE 1 :

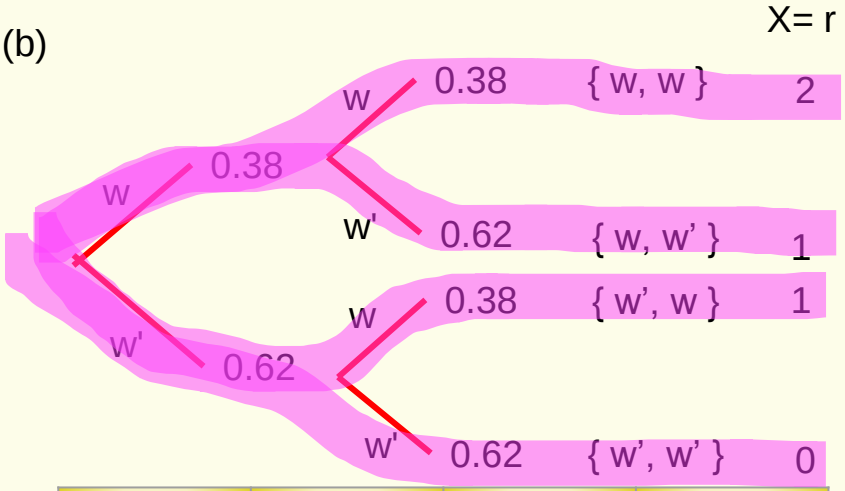
**In 2016, it was found that 38% of the cars purchased by Malaysians were white. If two buyers were selected at random and X represents the number of white car's buyers**

(a) state the set of X,

(b) draw a tree diagram and determine the probability distribution of X.

(a) X = { 0, 1, 2 }

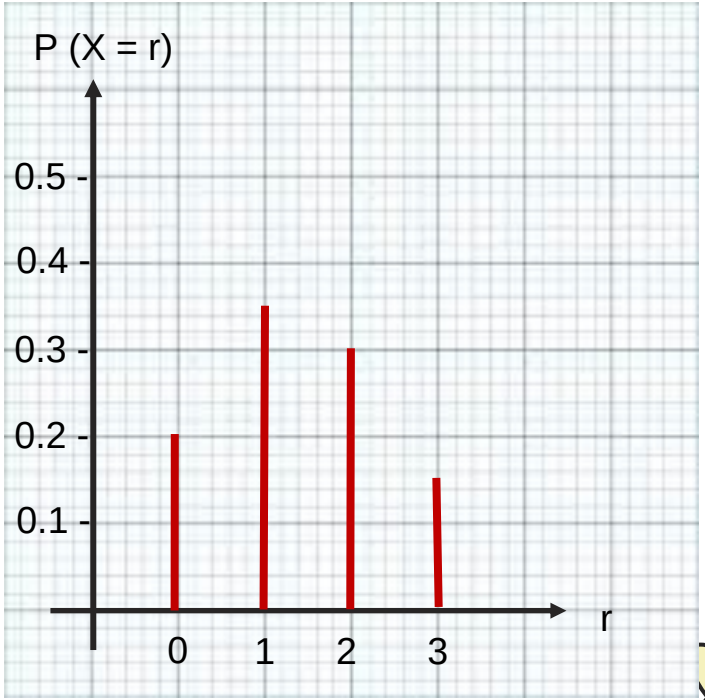
(b)



# Table and graph of probability distribution for discrete random variable

Given X = {0, 1, 2, 3} is a discrete random variable that represents the number of computers in an office together with their respective probability functions as shown in the table below, Draw the probability distribution graph for X.

| X = r | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P ( X = r ) | 0.2 | 0.35 | 0.3 | 0.15 |



# BINOMIAL DISTRIBUTION

02

03

# LEARNING STANDARDS

- Describe the meaning of binomial distribution.
- Determine the probability of an event for binomial distribution.
- Interpret information, construct table and draw graph of binomial distribution.
- Determine and describe the value of mean, variance and standard deviation for a binomial distribution
- Solve problems involving binomial distributions

# Bernoulli Trials

## The characteristics of Bernoulli trials are as follows:

- There are only **two possible outcomes**, namely 'success' and 'failure'.
- The chances of 'success' are **always the same in every trial**.
- If the probability of 'success' is given by p, then the probability of 'failure' is given by q; (1 – p).

# Binomial distribution

**An experiment which is made up of n similar Bernoulli trials is known as a binomial experiment.**

$$X \sim B(n, p)$$

n – number of trials

p – probability of success

# Probability of an event for binomial distribution

The following formulae may be helpful in answering the questions. The symbols given are the ones commonly used.

1. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
2. $\log_a b=\frac{\log_c b}{\log_c a}$
3. $T_n=a+(n-1)d$
4. $T_n=ar^{n-1}$
5. $S_n=\frac{n}{2}[2a+(n-1)d]$
6. $S_n=\frac{a(r^n-1)}{r-1}=\frac{a(1-r^n)}{1-r},r\neq 1$
7. $Z=\frac{x-\mu}{\sigma}$
8. $P(X=r)={}^nC_r p^r q^{n-r}, p+q=1$
9. ${}^nP_r=\frac{n!}{(n-r)!}$
10. ${}^nC_r=\frac{n!}{(n-r)!r!}$
11. $I=\frac{Q}{Q_0}\times 100$
12. $\bar{I}=\frac{\sum W_i I_i}{\sum W_i}$
13. $\sin^2 A+\cos^2 A=1$
    $sin^2 A+kos^2 A=1$
14. $\sec^2 A=1+\tan^2 A$
    $sek^2 A=1+tan^2 A$
15. $\operatorname{cosec}^2 A=1+\cot^2 A$
    $kosek^2 A=1+kot^2 A$
16. $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$
    $sin(A\pm B)=sinA\ kos\ B\pm kosA\ sin\ B$
17. $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$
    $kos(A\pm B)=kos\ A\ kosB\mp sin\ A\ sin\ B$
18. $\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$
19. $\sin 2A=2\sin A\cos A$
    $sin\ 2A=2\ sin\ A\ kos\ A$
20. $\cos 2A=\cos^2 A-\sin^2 A$
    $=2\cos^2 A-1$
    $=1-2\sin^2 A$
    $kos\ 2A=kos^2 A-sin^2 A$
    $=2\ kos^2 A-1$
    $=1-2sin^2 A$
21. $\tan 2A=\frac{2\tan A}{1-\tan^2 A}$
22. $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
23. Area of triangle / *Luas segi tiga*
    $=\frac{1}{2}ab\sin C$

$$P(X=r)={}^nC_r p^r q^{n-r}, p+q=1$$

- r = represents the number of ‘success’
- n = independent trials of an experiment
- p = the probability of ‘success’
- q = 1 – p as the probability of ‘failure’

# Probability of an event for binomial distribution

## EXAMPLE 2 :

In an examination, 70% of the students passed. If a sample of 8 students is randomly selected, find the probability that 6 students from the sample passed the examination.

$$P(X=r) = {}^{n}C_{r}\,p^{r}q^{n-r},\ p+q=1$$

Info: $p = 0.7$ , $q = 0.3$ , $n = 8$

Question need: $P(X = 6)$=?

Solution:

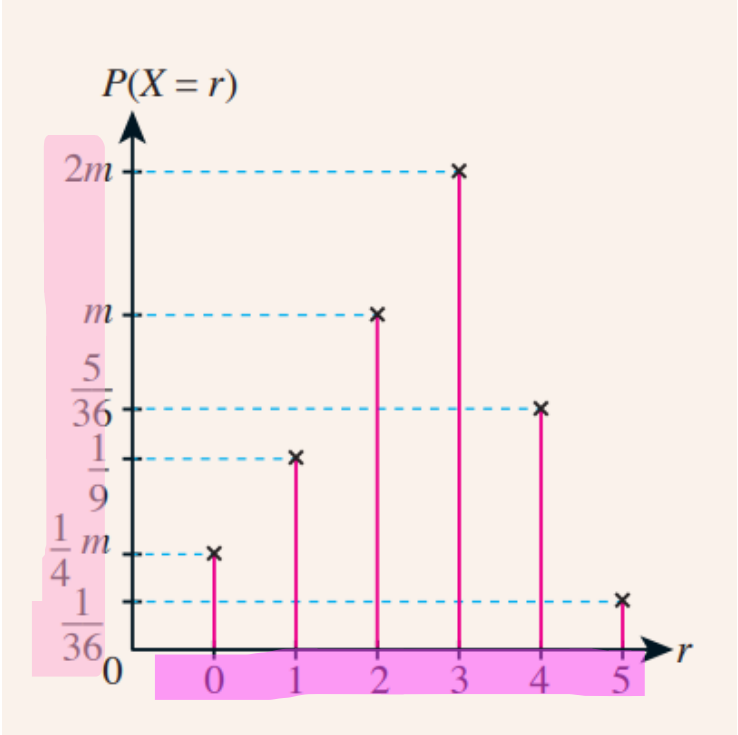
$$P(X = 6) = ({}^{8}C_{6})(0.7^{6})(0.3^{2})$$

$$= 0.2965$$

# Constructing table, drawing graph and INTERPRETING information of binomial distribution

## EXAMPLE 3:

The diagram on the right shows a binomial distribution graph for the discrete random variable X.




(a) State all the possible outcomes of X.

Answer: X = { 0, 1, 2, 3, 4, 5 }

(b) Find the value of m from the graph.

**P(X =0) + P(X =1) + P(X =2) + P(X =3) + P(X =4) + P(X = 5) = 1**

$$\frac{1}{4}m + \frac{1}{9} + m + 2m + \frac{5}{36} + \frac{1}{36} = 1$$

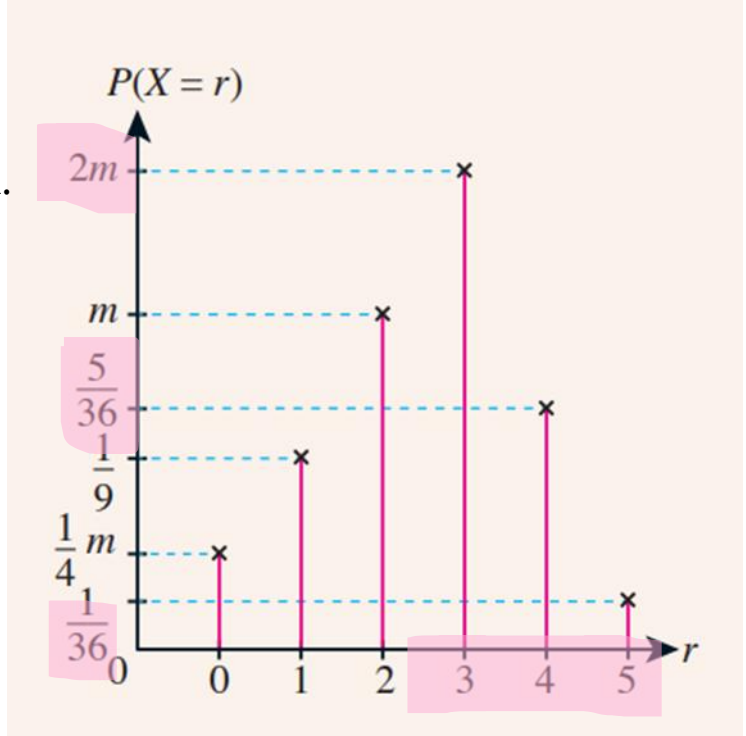
$$\frac{13}{4}m = \frac{13}{18}$$

$$m = \frac{2}{9}$$

# Constructing table, drawing graph and INTERPRETING information of binomial distribution

## EXAMPLE 3:

The diagram on the right shows a binomial distribution graph for the discrete random variable X.




(c) Find the percentage for P(X > 2).

$$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5)$$

$$= \frac{4}{9} + \frac{5}{36} + \frac{1}{36}$$

$$= \frac{11}{18} \quad // 0.6111$$

$$\therefore Percentage\ for\ P(X > 2) = 0.6111 \times 100\%$$

$$= 61.11\%$$

# The value of mean, variance and standard deviation for a binomial distribution.

Mean, $\mu = np$

Variance, $\sigma^2 = npq$

Standard deviation, $\sigma = \sqrt{npq}$

- **n = independent trials of an experiment**
- **p = the probability of 'success'**
- **q = 1 – p, as the probability of 'failure'**

# Solving problems involving binomial distributions

## EXAMPLE 4:

The probability that Ali scored a goal from a penalty kick in a soccer practice is $t$. Ali attempts $n$ penalty kicks and the number of goals is recorded. Given that the mean and the standard deviation of the number of goals scored are 60 and 6 respectively, find the value of $t$ and of $n$.

Mean, $\mu = np$

Standard deviation, $\sigma = \sqrt{npq}$

- Info: $p = t$ $\mu = 60$ $\sigma = 6$
- Question need: $t = ?$ , $n = ?$
- Solution: $60 = nt$------ eq1

  $6 = \sqrt{ntq}$----- eq 2

Substitute eq1 to eq 2:

$6 = \sqrt{(60)q}$

$36 = 60\,q$

$0.6 = q$

$t = p = 0.4$

$nt = 60$

$n\,(0.4) = 60$

$n = 150$

# Solve problem related to Binomial

## EXAMPLE 5:

$$P(X=r) = {}^{n}C_{r}\,p^{r}q^{n-r},\ p+q=1$$

Farah made 5 attempts in an archery practice. The probability that Farah strikes the target in an attempt is 0.7. It is given that $X$ is a discrete random variable that represents the number of times Farah strikes the target.

(a) List all the elements of $X$.

(b) Calculate the probability that Farah strikes the target at least 2 times.

- Info: n = 5, p = 0.7, q = 0.3
- Question need: a) X = { ? }
  b) P(X ≥ 2)
- Solution:

a) X = { 0, 1, 2, 3, 4, 5 }

b) P(X ≥ 2)

= P(X= 2)+ P(X=3)+ P(X=4)+ P(X=5)

= 1 – P(X = 0) – P(X = 1)

= $1 - ({}^{5}c_{0})(0.7^{0})(0.3^{5}) - ({}^{5}c_{1})(0.7^{1})(0.3^{4})$

= 1 – 0.00243 – 0.02835

= 0.96922

# COMMON TIPS FOR BINOMIAL DISTRIBUTION

$^nC_0 = 1$ $^nC_n = 1$ $^nC_1 = n$ $a^0 = 1$

Notes:

- Less than ( $<$ )
- At most// not more than// Does not exceed ( $\leq$ )
- More than ( $>$ )
- At least// not less than ( $\geq$ )

# NORMAL DISTRIBUTION

03

# LEARNING STANDARDS

- Investigate and describe the properties of normal distribution graph.
- Describe the meaning of standard normal distribution.
- Determine and interprete standard score, Z.
- Determine the probability of an event for normal distribution.
- Solve problems involving normal distributions.

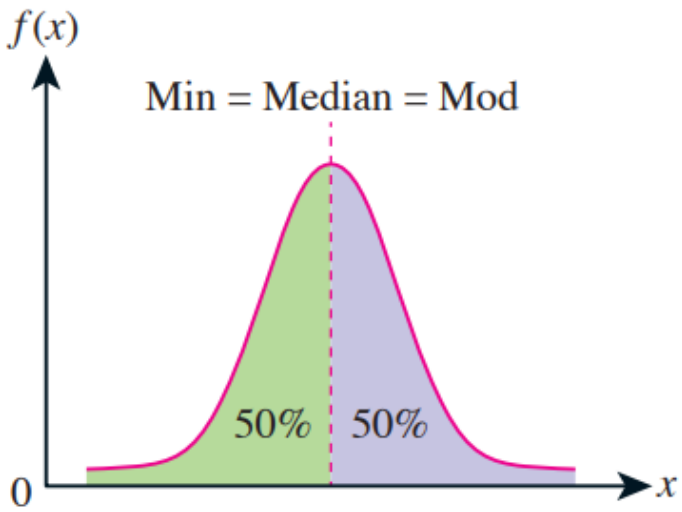
# NORMAL DISTRIBUTION

$$X \sim N(\mu, \sigma^2)$$

$\mu$ – mean

$\sigma^2$ - variance

# The properties of normal distribution graph




## Important features of a normal distribution function graph are:

- The curve is **bell-shaped** and is **symmetrical** about a vertical line that **passes through the mean, μ .**
- The curve has a **maximum value at** the axis of symmetry, **X = μ .**
- The mean, **μ divides the region** under the graph **into two equal parts**
- Both ends of the curve extend indefinitely without touching the x-axis
- **The total area under the graph** is equal to the **total probability** of all outcomes, **that is, 1 unit$^2$ .**

# INTERPRET NORMAL GRAPH

## EXAMPLE 6 :

The diagram on the right shows a normal distribution graph for a continuous random variable X

(a) State the mean of X.

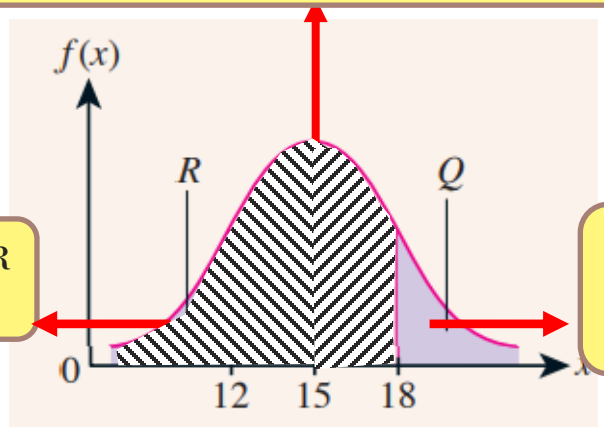
(b) Express the shaded regions Q and R in probability notations.

(c) If P(X<18) = 0.7635, find

i) P(X > 18)

ii) P(15 < X < 18)

- The mean, μ divides the region under the graph into two equal parts.
- (a) mean of X = 15




(b) Region R
P( X < 12)

(b) Region Q
P( X > 18)
=0.2635

(c) Info : P(X<18) = 0.7635

Question need: ii) P(15 < X <18)

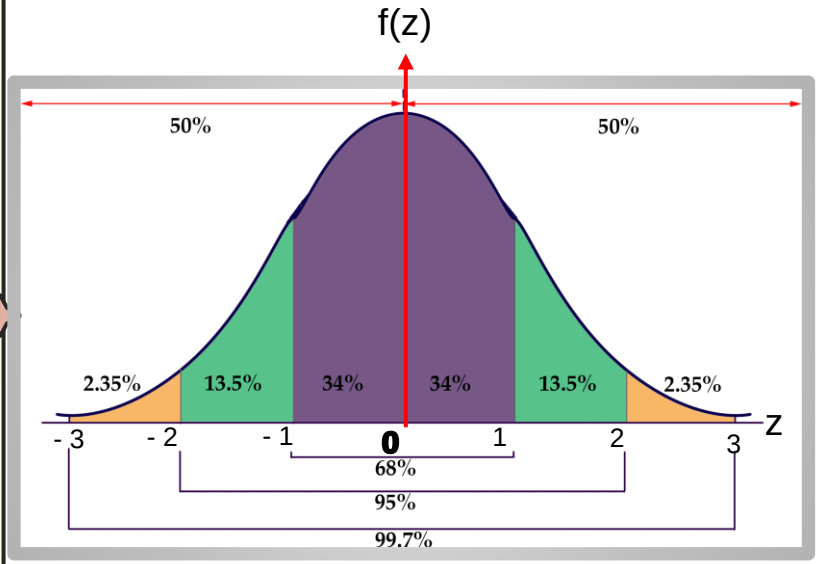
Solution: ii) P(15 < X <18) = P(X >15) – P(X >18)
= 0.5 – 0.2365
= 0.2635

# STANDARD NORMAL DISTRIBUTION

$$Z = \frac{X - \mu}{\sigma}$$

Where $Z \sim N(0, 1)$

# STANDARD NORMAL DISTRIBUTION GRAPH




## Important features of a standard normal distribution function graph:

- 68% of the data lies within the standard deviation ±1 from the mean.
- 95% of the data lies within the standard deviation ±2 from the mean.
- 99.7% of the data lies within the standard deviation ±3 from the mean.

# Determining and interpreting standard score, Z // X value

$$Z = \frac{X - \mu}{\sigma}$$

## EXAMPLE 7 :

A continuous random variable X is normally distributed with mean, $\mu = 24$ and a standard deviation, $\sigma = 6$. Find the z-score if X = 19.5

Info: $\mu = 24$ $\sigma = 6$ x = 19.5

Question need: z = ?

Solution :

$$Z = \frac{19.5 - 24}{6}$$

$$= -0.75$$

## EXAMPLE 8:

X is a continuous random variable that is normally distributed, such that X ~ N(500, 169). Find the value of X if the z-score is 1.35.

Info: $\mu = 500$ $\sigma = 13$ z = 1.35

Question need: x = ?

Solution :

$$1.35 = \frac{x - 500}{13}$$

$$x = 517.55$$

# Determining the probability of an event for normal distribution

$$P(X < a) = P\left(Z < \frac{a-\mu}{\sigma}\right)$$

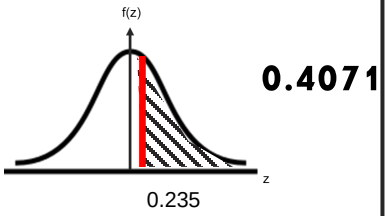
$$P(X > a) = P\left(Z > \frac{a-\mu}{\sigma}\right)$$

$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

# Find the probability of normal distribution

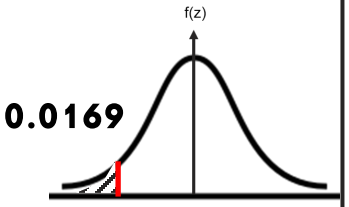
**P(Z ≥0.235)**

= 0.4090 – 0.0019
= 0.4071




**P(Z < −2.122)**

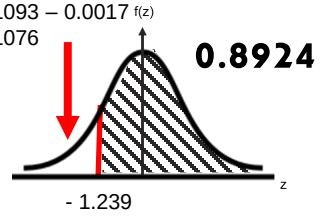
= 0.0170 – 0.0001
= 0.0169




**P(Z ≥−1.239)**

= 1 - 0.1076
0.8924

= 0.1093 – 0.0017
= 0.1076

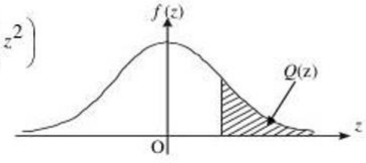



**THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)**
***KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)***

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0..0475 | 0.0465 | 0.0455 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right)$$

$$Q(z) = \int_z^\infty f(z)\,dz$$




Example / *Contoh*:

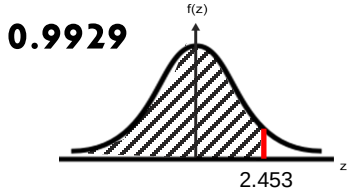
If $X \sim N(0, 1)$, then $P(X > k) = Q(k)$
*Jika* $X \sim N(0, 1)$, *maka* $P(X > k) = Q(k)$

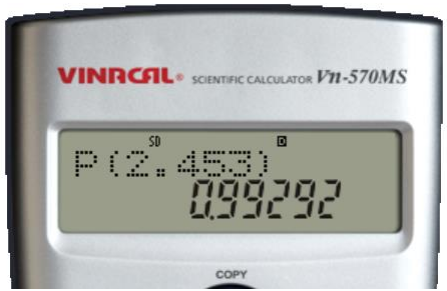
# Find the probability of normal distribution

**P(Z ≤ 2.453)**

= P(2.453)

= 0.9929

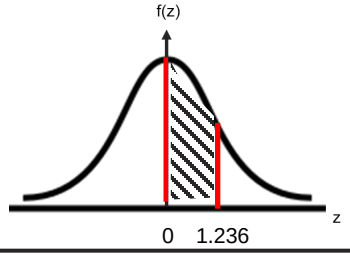


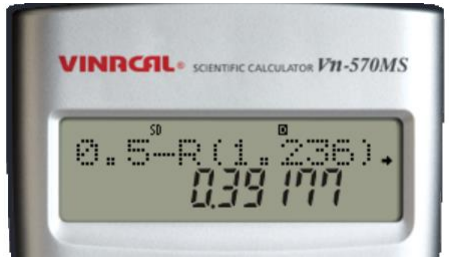




**P(0 ≤ Z ≤ 1.236)**

= 0.5 - R(1.236)

= **0.3918**

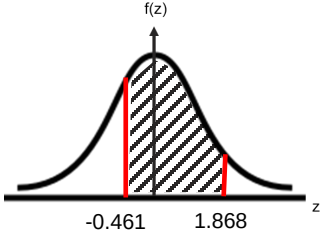


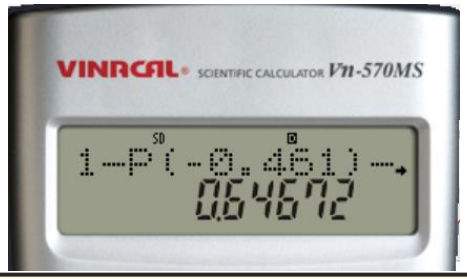




**P(−0.461 ≤ Z ≤ 1.868)**

= 1 – P(-0.461) - R(1.868)

= **0.6467**



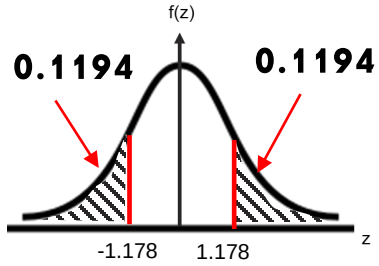



# Find the probability of normal distribution

$P(|Z| \geq 1.178)$

$= 2[0.1194]$

$= 0.2388$

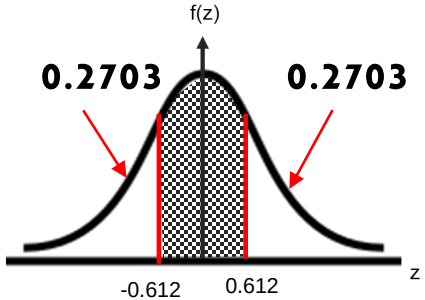




$P(|Z| \leq 0.612)$

$= 1 - 2(0.2703)$

$= 0.4594$




# Determining the probability of an event for normal distribution

## EXAMPLE 9 :

The heights of Form 1 pupils in a certain school are normally distributed with a mean of 145 cm and a standard deviation of 10 cm. If a pupil is randomly selected from that group, find the probability that the pupil's height is at least 140 cm.

Info: $\mu = 145$ $\sigma = 10$

Question need: $\mathrm{P}(X \geq 140)$

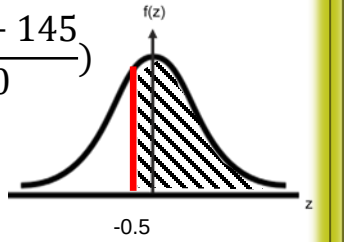
Solution: $\mathrm{P}(X \geq 140)$

$$= \mathrm{P}\left(Z \geq \frac{140 - 145}{10}\right)$$

$$= \mathrm{P}(Z \geq -0.5)$$

$$= R(-0.5)$$

$$= 0.6915$$



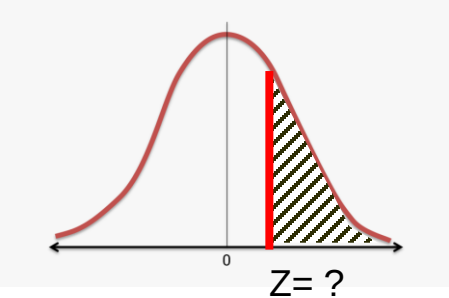
**THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)**

***KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)***

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.1 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | | | | | | | | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| | | | | 0.00990 | 0.00964 | 0.00939 | 0.00914 | | | | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| | | | | | | | | 0.00889 | 0.00866 | 0.00842 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 16 | 21 |
| 2.4 | 0.00820 | 0.00798 | 0.00776 | 0.00755 | 0.00734 | | | | | | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| | | | | | | 0.00714 | 0.00695 | 0.00676 | 0.00657 | 0.00639 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 2.5 | 0.00621 | 0.00604 | 0.00587 | 0.00570 | 0.00554 | 0.00539 | 0.00523 | 0.00508 | 0.00494 | 0.00480 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 2.6 | 0.00466 | 0.00453 | 0.00440 | 0.00427 | 0.00415 | 0.00402 | 0.00391 | 0.00379 | 0.00368 | 0.00357 | 1 | 2 | 3 | 5 | 6 | 7 | 9 | 9 | 10 |
| 2.7 | 0.00347 | 0.00336 | 0.00326 | 0.00317 | 0.00307 | 0.00298 | 0.00289 | 0.00280 | 0.00272 | 0.00264 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2.8 | 0.00256 | 0.00248 | 0.00240 | 0.00233 | 0.00226 | 0.00219 | 0.00212 | 0.00205 | 0.00199 | 0.00193 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 2.9 | 0.00187 | 0.00181 | 0.00175 | 0.00169 | 0.00164 | 0.00159 | 0.00154 | 0.00149 | 0.00144 | 0.00139 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 3.0 | 0.00135 | 0.00131 | 0.00126 | 0.00122 | 0.00118 | 0.00114 | 0.00111 | 0.00107 | 0.00104 | 0.00100 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |

# FIND THE Z SCORE IF THE PROBABILITY OF NORMAL DISTRIBUTION IS GIVEN.

$$P(Z \geq a) = 0.3851$$




**Z = 0.292**

$$\therefore a = 0.292$$

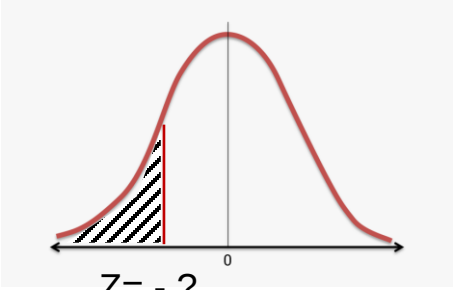
**THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)**

***KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)***

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | | | | | | | | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| | | | | 0.00990 | 0.00964 | 0.00939 | 0.00914 | | | | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| | | | | | | | | 0.00889 | 0.00866 | 0.00842 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 16 | 21 |
| 2.4 | 0.00820 | 0.00798 | 0.00776 | 0.00755 | 0.00734 | | | | | | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| | | | | | | 0.00714 | 0.00695 | 0.00676 | 0.00657 | 0.00639 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 2.5 | 0.00621 | 0.00604 | 0.00587 | 0.00570 | 0.00554 | 0.00539 | 0.00523 | 0.00508 | 0.00494 | 0.00480 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 2.6 | 0.00466 | 0.00453 | 0.00440 | 0.00427 | 0.00415 | 0.00402 | 0.00391 | 0.00379 | 0.00368 | 0.00357 | 1 | 2 | 3 | 5 | 6 | 7 | 9 | 9 | 10 |
| 2.7 | 0.00347 | 0.00336 | 0.00326 | 0.00317 | 0.00307 | 0.00298 | 0.00289 | 0.00280 | 0.00272 | 0.00264 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2.8 | 0.00256 | 0.00248 | 0.00240 | 0.00233 | 0.00226 | 0.00219 | 0.00212 | 0.00205 | 0.00199 | 0.00193 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 2.9 | 0.00187 | 0.00181 | 0.00175 | 0.00169 | 0.00164 | 0.00159 | 0.00154 | 0.00149 | 0.00144 | 0.00139 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 3.0 | 0.00135 | 0.00131 | 0.00126 | 0.00122 | 0.00118 | 0.00114 | 0.00111 | 0.00107 | 0.00104 | 0.00100 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |

# FIND THE Z SCORE IF THE PROBABILITY OF NORMAL DISTRIBUTION IS GIVEN.

$$P(Z < a) = 0.3851$$




Z= - ?

**Z = - 0.292**

$$\therefore a = -0.292$$

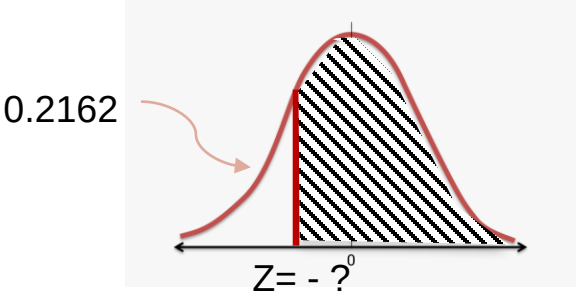
**THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)**

***KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)***

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | | | | | | | | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| | | | | 0.00990 | 0.00964 | 0.00939 | 0.00914 | | | | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| | | | | | | | | 0.00889 | 0.00866 | 0.00842 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 16 | 21 |
| 2.4 | 0.00820 | 0.00798 | 0.00776 | 0.00755 | 0.00734 | | | | | | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| | | | | | | 0.00714 | 0.00695 | 0.00676 | 0.00657 | 0.00639 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 2.5 | 0.00621 | 0.00604 | 0.00587 | 0.00570 | 0.00554 | 0.00539 | 0.00523 | 0.00508 | 0.00494 | 0.00480 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 2.6 | 0.00466 | 0.00453 | 0.00440 | 0.00427 | 0.00415 | 0.00402 | 0.00391 | 0.00379 | 0.00368 | 0.00357 | 1 | 2 | 3 | 5 | 6 | 7 | 9 | 9 | 10 |
| 2.7 | 0.00347 | 0.00336 | 0.00326 | 0.00317 | 0.00307 | 0.00298 | 0.00289 | 0.00280 | 0.00272 | 0.00264 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2.8 | 0.00256 | 0.00248 | 0.00240 | 0.00233 | 0.00226 | 0.00219 | 0.00212 | 0.00205 | 0.00199 | 0.00193 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 2.9 | 0.00187 | 0.00181 | 0.00175 | 0.00169 | 0.00164 | 0.00159 | 0.00154 | 0.00149 | 0.00144 | 0.00139 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 3.0 | 0.00135 | 0.00131 | 0.00126 | 0.00122 | 0.00118 | 0.00114 | 0.00111 | 0.00107 | 0.00104 | 0.00100 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |

# FIND THE Z SCORE IF THE PROBABILITY OF NORMAL DISTRIBUTION IS GIVEN.

$$P(Z \geq a) = 0.7838$$




**Z = - 0.785**

$$\therefore a = -0.785$$

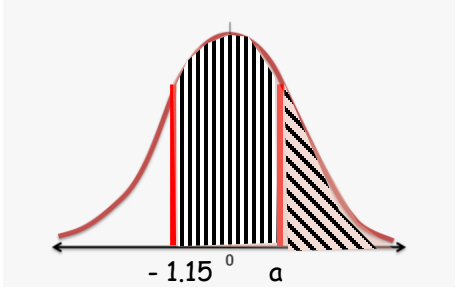
**THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)**

***KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)***

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | | | | | | | | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| | | | | 0.00990 | 0.00964 | 0.00939 | 0.00914 | | | | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| | | | | | | | | 0.00889 | 0.00866 | 0.00842 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 16 | 21 |
| 2.4 | 0.00820 | 0.00798 | 0.00776 | 0.00755 | 0.00734 | | | | | | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| | | | | | | 0.00714 | 0.00695 | 0.00676 | 0.00657 | 0.00639 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 2.5 | 0.00621 | 0.00604 | 0.00587 | 0.00570 | 0.00554 | 0.00539 | 0.00523 | 0.00508 | 0.00494 | 0.00480 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 2.6 | 0.00466 | 0.00453 | 0.00440 | 0.00427 | 0.00415 | 0.00402 | 0.00391 | 0.00379 | 0.00368 | 0.00357 | 1 | 2 | 3 | 5 | 6 | 7 | 9 | 9 | 10 |
| 2.7 | 0.00347 | 0.00336 | 0.00326 | 0.00317 | 0.00307 | 0.00298 | 0.00289 | 0.00280 | 0.00272 | 0.00264 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2.8 | 0.00256 | 0.00248 | 0.00240 | 0.00233 | 0.00226 | 0.00219 | 0.00212 | 0.00205 | 0.00199 | 0.00193 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 2.9 | 0.00187 | 0.00181 | 0.00175 | 0.00169 | 0.00164 | 0.00159 | 0.00154 | 0.00149 | 0.00144 | 0.00139 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 3.0 | 0.00135 | 0.00131 | 0.00126 | 0.00122 | 0.00118 | 0.00114 | 0.00111 | 0.00107 | 0.00104 | 0.00100 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |

# FIND THE Z SCORE IF THE PROBABILITY OF NORMAL DISTRIBUTION IS GIVEN.

$$P(-1.15 < Z < a) = 0.7838$$




$P(Z > -1.15)$ $-P(Z > a)$ = 0.7838

R(-1.15) $-P(Z > a)$ = 0.7838

$0.8749 - 0.7838 = P(Z > a)$

$P(Z > a) = 0.0911$

**$a$ = 1.334**

# Solving problems involving normal distributions

## EXAMPLE 10 :

The masses of papayas produced in an orchard have a normal distribution with a mean of 840 g and a standard deviation of 24 g. The papayas with masses between 812 g and 882 g will be exported overseas while papayas that weigh 812g or less will be sold at the local market. Find

(a) the probability that a papaya chosen at random to be exported overseas,

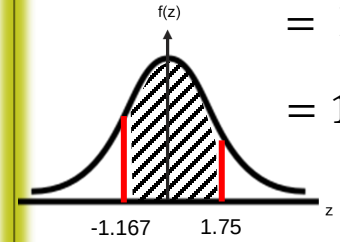
(b) the number of papayas which are not exported overseas and not sold in the local market if the orchard produces 2 500 papayas.

(a) Info: $\mu = 840$ $\sigma = 24$

Question need: $P(812 \leq X \leq 882)$

Solution:

$$= P\left(\frac{812-840}{24} \leq Z \leq \frac{882-840}{24}\right)$$




$$= P(-1.167 \leq Z \leq 1.75)$$

$$= 1 - P(-1.167) - R(1.75)$$

$$= 0.83833$$

# Solving problems involving normal distributions

## EXAMPLE 10 :

The masses of papayas produced in an orchard have a normal distribution with a mean of 840 g and a standard deviation of 24 g. The papayas with masses between 812 g and 882 g will be exported overseas while papayas that weigh 812g or less will be sold at the local market. Find

(a) the probability that a papaya chosen at random to be exported overseas,

(b) the number of papayas which are not exported overseas and not sold in the local market if the orchard produces 2 500 papayas.

(b)

Info: $\mu = 840$ | $\sigma = 24$ | Total = 2500

Question need: $P(X \geq 882) = ?$ ; $n(X \geq 882) = ?$

Solution:

$$P(X \geq 882)$$

$$= P\left(Z \geq \frac{882-840}{24}\right)$$

$$= P(Z \geq 1.75)$$

$$= 0.0401$$

$$P(x) = \frac{n(x)}{n(S)}$$

$$\frac{n(not\ sold)}{2500} = 0.0401$$

$$= 100.25$$

$$n(not\ sold) = 100$$

# Solving problems involving normal distributions

## EXAMPLE 11 :

The masses of babies born in a hospital are normally distributed with a mean of 3.1 kg and a standard deviation of 0.3 kg. If 25% of babies born in that hospital are categorised as underweight, find the maximum mass for this category.

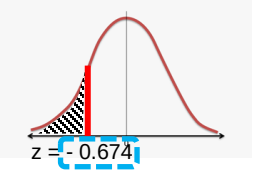
THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)
*KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)*

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |

Info: $\mu = 3.1$ $\sigma = 0.3$ $P(X \leq m) = 0.25$

Question need: $m = ?$

Solution: $P(X \leq m) = 0.25$

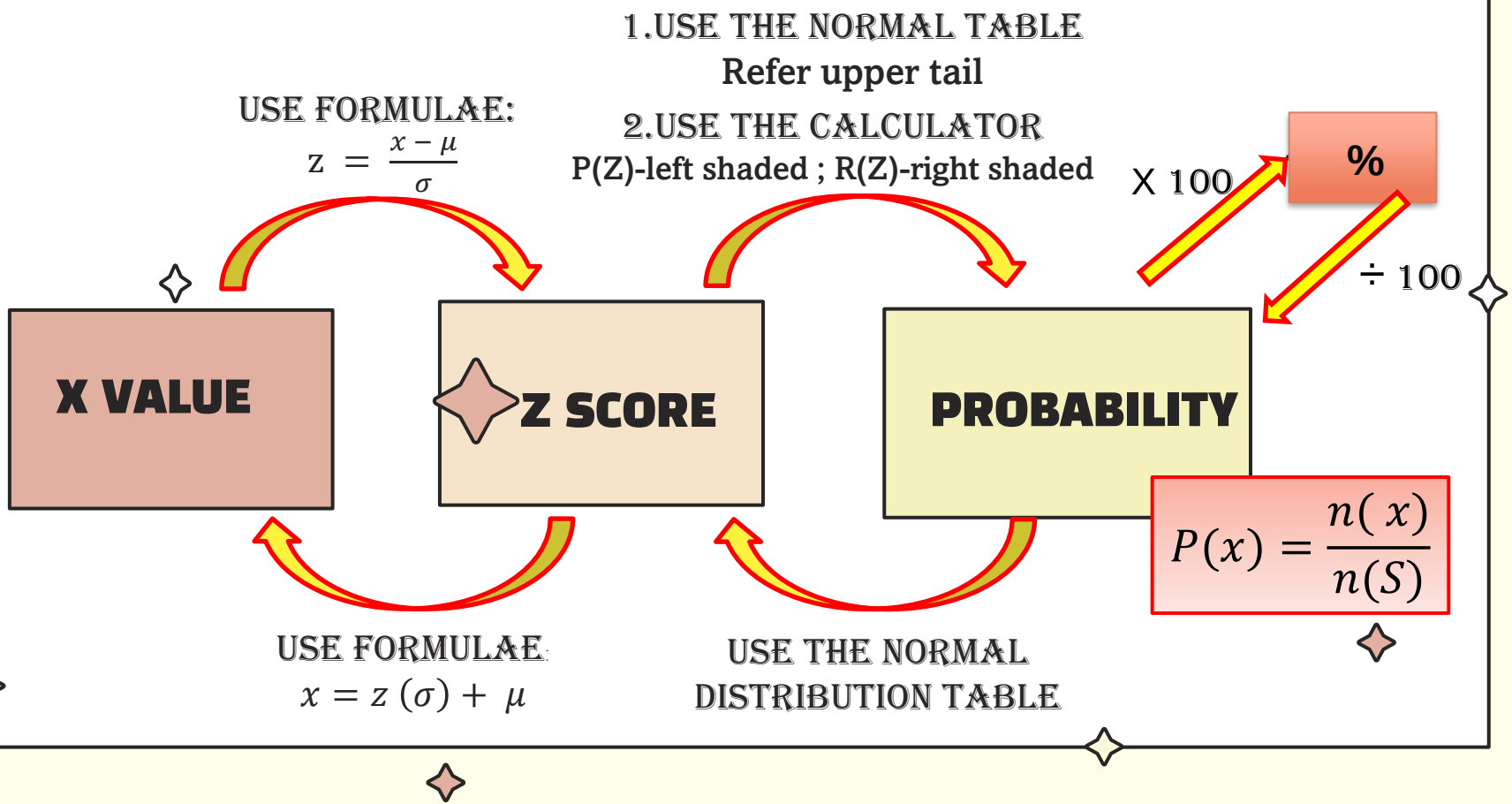



$$= P\left(Z \leq \frac{m - 3.1}{0.3}\right) = 0.25$$

$$\therefore \frac{m - 3.1}{0.3} = -0.674$$

$$m = 2.8978$$

# RECALL WHAT WE HAVE GO THROUGH



# SPM FORMAT QUESTION

# EXAMPLE 12 : SPM 2014 PAPER 2

(a) A Survey is carried out about a scout in a school. It is found that the mean of the number of scouts is 315, the variance is 126 and the probability that a student participate in scout is p.

(i) Find the value of p.

(ii) If 8 students from the school are chosen at random, find the probability that more than 5 students participate in scout.

[5 marks]

(b) The mass of the scout members in the school follows a normal distribution with a mean of 48 kg and a standard deviation of 5.8 kg. Find

(i) the probability that a member chosen at random from the group has a mass less than 45 kg.

(ii) the value of m, if 25% of the scout members have mass more than m kg.

[5 marks]

(a) (i)

Info: $mean = 315$ | $variance = 126$ | $p(scout) = p$

Question Need: $p = ?$

Mean, $\mu = np$ Variance, $\sigma^2 = npq$

$315 = np$------ eq1 $126 = npq$ ----- eq 2 ✓1

Substitute eq 1 to eq 2:

$126 = (315)q$ ✓1

$q = 0.4$ $\therefore p = 0.6$ ✓1

(a) (ii)

Info: $p = 0.6$ | $q = 0.4$ | $n = 8$

Question Need: $P(X > 5) = ?$

Solution: $P(X > 5) = P(X=6) + P(X=7) + P(X=8)$ ✓1

$= ({}^8C_6)(0.6^6)(0.4^2) + ({}^8C_7)(0.6^7)(0.4^1) + ({}^8C_8)(0.6^8)(0.4^0)$

$= 0.2090 + 0.0896 + 0.0168$

$= 0.3154$ ✓1

# EXAMPLE 12 :

**THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)**
***KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)***

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |

[5 marks]

(b) The mass of the scout members in the school follows a normal distribution with a mean of 48 kg and a standard deviation of 5.8 kg. Find

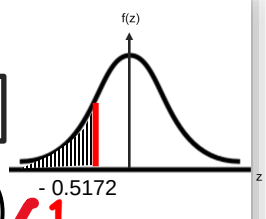
(i) the probability that a member chosen at random from the group has a mass less than 45 kg.

(ii) the value of m, if 25% of the scout members have mass more than m kg.

[5 marks]

(b) (i) Info: $\mu = 48$ $\sigma = 5.8$

Question Need: $P(X < 45) = ?$




Solution:

$$P(X < 45) = P\left(Z < \frac{45-48}{5.8}\right)$$ ✓1

$$= P(Z < -0.5172)$$

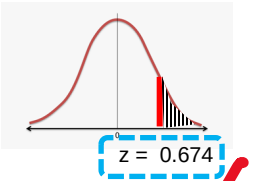
$$= P(-0.5172)$$

$$= 0.3025$$ ✓1

(b) (ii) Info: $\mu = 48$ $\sigma = 5.8$

Question Need: $P(X > m) = 0.25$; m = ?

Solution: $P(X > m) = 0.25$



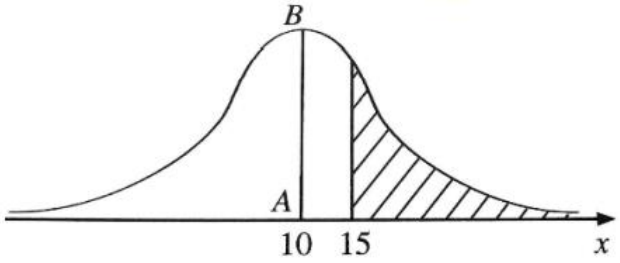

$$P\left(Z > \frac{m-48}{5.8}\right) = 0.25$$ ✓1

$$\frac{m-48}{5.8} = 0.674$$ ✓1

$$m = 51.71$$ ✓1

# EXAMPLE 13 : SPM 2017 PAPER 1

Diagram shows a probability distribution graph for a random variable X, $X \sim N(\mu, \sigma^2)$. It is given that AB is the axis of symmetry of the graph.

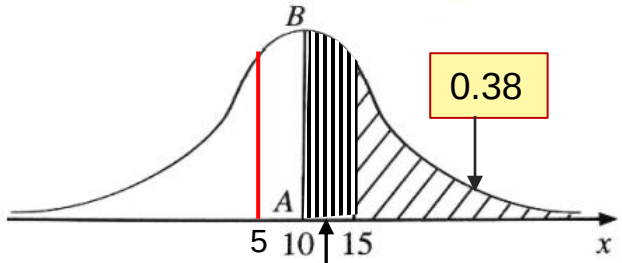



(a) State the value of μ.

(b) If the area of shaded region is 0.38, state the value of $P(5 \leq X \leq 15)$.

[2 marks]

- The mean, μ divides the region under the graph into two equal parts.




(a) $\mu = 10$ ✓1

(b) Info: $P(X > 15) = 0.38$

Question Need: $P(5 \leq X \leq 15)$

$P(5 \leq X \leq 15)$

$= (0.5 - 0.38) \times 2$

$= 0.24$ ✓1

# EXAMPLE 14 : SPM 2016 PAPER 2

(a) It is found that 20% of the students from Kampung Aman walk to school. If 8 students from Kampung Aman are chosen at random, find the probability that exactly 3 of them walk to school. (2 marks)

(b) The mass of pineapples harvested from a farm follows a normal distribution with mean of 2 kg and standard deviation of m kg. It is given that 15.87% of the pineapples have a mass more than 2.5kg.

(i) Calculate the value of m.

(ii) Given the number of pineapples harvested from the farm is 1320, find the number of pineapples that have the mass between 1.0 kg and 2.5 kg.

. (8 marks)

(a) Info: $p = 0.2$ $q = 0.8$ $n = 8$

Question Need: $P(X=3) = ?$

Solution: $P(X = 3) = ({}^{8}c_{3})(0.2^{3})(0.8^{5})$ ✓1

$= 0.1468$ ✓1

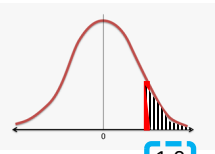
(b) (i)

Info: $\mu = 2$ $\sigma = m$ $P(X > 2.5) = 0.1587$

Question Need: $m = ?$

Solution: $P(X > 2.5) = 0.1587$

$P\left(Z > \frac{2.5 - 2}{m}\right) = 0.1587$




✓1

$\frac{2.5 - 2}{m} = 1.0$ ✓1

m = 0.5 ✓1

# EXAMPLE 14:

(a) It is found that 20% of the students from Kampung Aman walk to school. If 8 students from Kampung Aman are chosen at random, find the probability that exactly 3 of them walk to school. (2 marks)

(b) The mass of pineapples harvested from a farm follows a normal distribution with mean of 2 kg and standard deviation of m kg. It is given that 15.87% of the pineapples have a mass more than 2.5kg.

(i) Calculate the value of m. (m = 0.5)

(ii) Given the number of pineapples harvested from the farm is 1320, find the number of pineapples that have the mass between 1.0 kg and 2.5 kg.

. (8 marks)

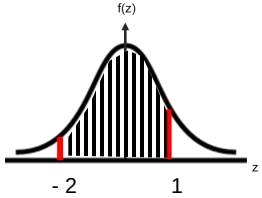
(b) (ii)

Info: $\mu = 2$ $\sigma = 0.5$ Total harvested = 1320

Question Need: $P(1.0 \le X \le 2.5) = ?$ $n(1.0 \le X \le 2.5) = ?$

Solution:

$\frac{n(X)}{n(S)} = P(X)$




$P(1.0 \le X \le 2.5)$

$= P\left(\frac{1.0-2}{0.5} \le Z \le \frac{2.5-2}{0.5}\right)$ ✓1

$= P(-2 \le Z \le 1)$

$= 1 - P(-2) - R(1)$

$= 0.8186$ ✓1

$\frac{n(1 \le mass \le 2.5)}{1320} = 0.8186$ ✓1

$n(1 \le mass \le 2.5) = 1080.55$ ✓1

$\therefore n(1 \le mass \le 2.5) = 1080 \text{ or } 1081$ ✓1

# EXAMPLE 15 : SPM 2019 PAPER 1

Diagram shows the normal distribution graph of the time for a school bus to arrive at a school.

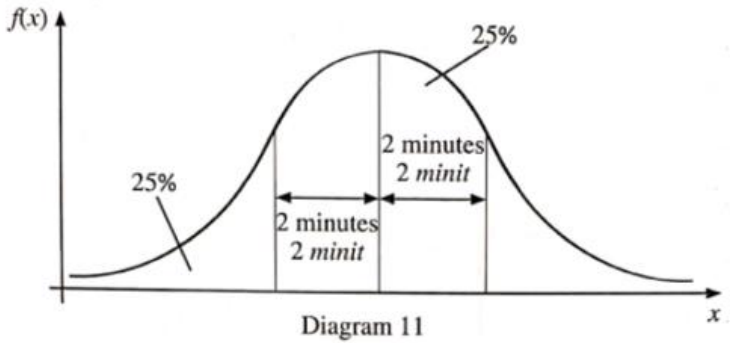



Diagram 11

a) Find the standard deviation

b) It is given that the mean time for the bus to arrive at the school is 7.15 a.m. Students are considered late if they arrived at 7.20 a.m. Lea takes the bus to the school.
Calculate the probability that Lea will be late.
Give your answer correct to three significant figures.

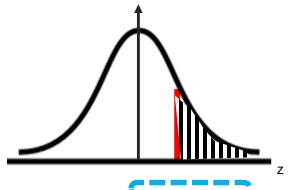
[4 marks]

**THE UPPER TAIL PROBABILITY Q(z) FOR THE NORMAL DISTRIBUTION N(0, 1)**
***KEBARANGKALIAN HUJUNG ATAS Q(z) BAGI TABURAN NORMAL N(0, 1)***

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Minus / *Tolak* | | | | | | | | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | | | | | | | | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| | | | | 0.00990 | 0.00964 | 0.00939 | 0.00914 | | | | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| | | | | | | | | 0.00889 | 0.00866 | 0.00842 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 16 | 21 |
| 2.4 | 0.00820 | 0.00798 | 0.00776 | 0.00755 | 0.00734 | | | | | | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| | | | | | | 0.00714 | 0.00695 | 0.00676 | 0.00657 | 0.00639 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 2.5 | 0.00621 | 0.00604 | 0.00587 | 0.00570 | 0.00554 | 0.00539 | 0.00523 | 0.00508 | 0.00494 | 0.00480 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 2.6 | 0.00466 | 0.00453 | 0.00440 | 0.00427 | 0.00415 | 0.00402 | 0.00391 | 0.00379 | 0.00368 | 0.00357 | 1 | 2 | 3 | 5 | 6 | 7 | 9 | 9 | 10 |
| 2.7 | 0.00347 | 0.00336 | 0.00326 | 0.00317 | 0.00307 | 0.00298 | 0.00289 | 0.00280 | 0.00272 | 0.00264 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2.8 | 0.00256 | 0.00248 | 0.00240 | 0.00233 | 0.00226 | 0.00219 | 0.00212 | 0.00205 | 0.00199 | 0.00193 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 2.9 | 0.00187 | 0.00181 | 0.00175 | 0.00169 | 0.00164 | 0.00159 | 0.00154 | 0.00149 | 0.00144 | 0.00139 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 3.0 | 0.00135 | 0.00131 | 0.00126 | 0.00122 | 0.00118 | 0.00114 | 0.00111 | 0.00107 | 0.00104 | 0.00100 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |

(a)

Info: $P(X \geq \mu + 2) = 0.25$




✓ 1

Question Need: $\sigma$ = ?

Solution: $P(X \geq \mu + 2) = 0.25$

$$P\left(Z \geq \frac{(\mu + 2) - \mu}{\sigma}\right) = 0.25$$

$$\frac{2}{\sigma} = 0.674$$

$$\sigma = 2.967 \quad \checkmark 1$$

(b)

If $X \geq 0720$ $(LATE)$

Info: $\mu = 0715$ $\sigma = 2.967$

Question Need: $P(X \geq 0720)$ $3.s.f$

Solution:

$$P(Z \geq \frac{0720 - 0715}{2.967}) \quad \checkmark 1$$

$$P(Z \geq 1.6852)$$

$$= 0.0460 \quad \checkmark 1$$

# EXAMPLE 16 : SPM 2019 PAPER 2

Diagram 4 shows a dart's target board at a dart game booth in a funfair.

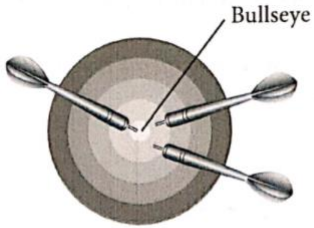



Diagram 4

The booth offers 3 darts per game. The customers have to pay RM5 to play a game. A toy bear will be given to customers who are able to hit the bullseye for the three darts' throws in a game. Bob is a dart player. By average, he hits the bullseye 7 times out of 10 darts thrown.

(a) Bob would play the game if he had at least 90% chance to win at least one toy bear by spending RM30.

By mathematical calculation, suggest to Bob whether he should play the game or otherwise. [7 *marks*]

(b) What is the minimum number of games that Bob needed so that he can get 4 toy bears? [3 *marks*]

(a) Info: $p = 0.7$ $q = 0.3$ $n = 3$

**If $P(X = 3)$; *Get a toy***

$P(X = 3) = ({}^{3}c_{3})(0.7^{3})(0.3^{0})$ ✓1

$= 0.343$ ✓1 (Probability Bob get a toy in 1 game)

By spending RM30, means Bob has to play 6 games:

$p = 0.343$ $q = 0.657$ $n = 6$ ✓1

**If $P(X \geq 1) \geq 0.9$ *so Bob would play***

$P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$ ✓1

$= 1 - P(X = 0)$

$= 1 - ({}^{6}c_{0})(0.343^{0})(0.657^{6})$ ✓1

$= 0.9196 \quad @ 91.96\%$ ✓1

∴ Suggest Bob to play the game because the probability to get at least 1 toys is more than 90%. ✓1

# EXAMPLE 16 : SPM 2019 PAPER 2

Diagram 4 shows a dart's target board at a dart game booth in a funfair.

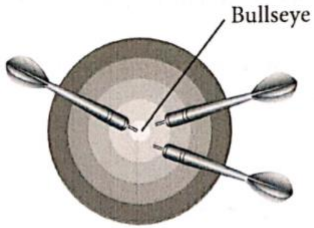



Diagram 4

The booth offers 3 darts per game. The customers have to pay RM5 to play a game. A toy bear will be given to customers who are able to hit the bullseye for the three darts' throws in a game. Bob is a dart player. By average, he hits the bullseye 7 times out of 10 darts thrown.

(*a*) Bob would play the game if he had at least 90% chance to win at least one toy bear by spending RM30.

By mathematical calculation, suggest to Bob whether he should play the game or otherwise. [7 *marks*]

(*b*) What is the minimum number of games that Bob needed so that he can get 4 toy bears? [3 *marks*]

(b) Info: $p = 0.343$ $Expected\ value, \mu = 4$

Question Need: $n = ?$

Mean, $\mu = np$

Solution: $4 = n\,(0.343)$ ✓1

$n = 11.66$ ✓1

∴ Bob should at least 12 games to get 4 toy bears. ✓1

Semoga kalian memperolehi ilmu yang bermanfaat... all the best for your coming SPMRSM & SPM

**Thank you**...

# Siri Jom Skor A+ Matematik Tambahan

SPM 2021

Sesi webinar *live* melalui Microsoft Teams

**13 Dis**
8.00 pm – 10.00 pm
Asniza Arshad
MRSM Tun Ghaffar Baba
Linear Programming

NEW
**15 Dis**
8.00 pm – 10.00 pm
Erwan Hazreen | MRSM Bentong
**Format 3472 dan Tips Skor!**

NEW
**17 Dis**
3.00 pm – 5.00 pm
Sahlawati Zakaria | MRSM Kuala Krai
**Soalan Mudah Skor!**

NEW
**17 Dis**
8.00 pm – 10.00 pm
Noraini Ismail | MRSM Transkrian
**HOTS Questions!**

Anjuran Unit Matematik
Bahagian Pendidikan Menengah MARA